Against this and the other features discussed in Section 3 must be weighed the disadvantages of the greater complexity of the electrohydrodynamic heat pipe, with its attendant reliability questions. A high voltage, low current power supply is required. The need for high voltage insulation in reases fabrication costs. Further, the long-term degradation of dielectric fluids in the presence of corona or intermittent arcing may be a significant factor.

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TURBULENT TRANSPORT COEFFICIENTS FOR COMPRESSIBLE HETEROGENEOUS MIXING

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I. INTRODUCTION

Within the past decade considerable amount of work has been done in the area of turbulent wakes and free mixing of coaxial jets due to the large interest in the wakes of high speed missiles and re-entry vehicles, various base injection techniques, cooled plasma jets, decaying exhaust plumes, supersonic combustion, etc. However, the problem is far from being solved yet. One of the main difficulties stems from the lack of information about the dependence of the turbulent transport coefficients on the flow properties. Although many models for the turbulent diffusivity [1] and viscosity [1–5] have been proposed, none has been found to be satisfactory so far. Therefore, an effort is made here to obtain theoretically the expressions for the turbulent transport coefficients on the basis of the general behavior characteristics of flow variables in the wake.

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II. THEORETICAL ANALYSIS

The governing equations for the coaxial turbulent mixing of two dissimilar gases, under the assumption of zero pressure gradients and no chemical reactions, are the following:

Conservation of Mass

$$\frac{\partial}{\partial x}(\rho u y^j) + \frac{\partial}{\partial y}(\rho v y^j) = 0 \quad (1)$$

Conservation of Species

$$\rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} = y^{-j} \frac{\partial}{\partial y}\left(\rho D y^j \frac{\partial Y_i}{\partial y}\right) \quad (2)$$

Conservation of Momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = y^{-j} \frac{\partial}{\partial y}\left(\mu y^j \frac{\partial u}{\partial y}\right) \quad (3)$$

Conservation of Energy

$$\rho u C_p \frac{\partial T}{\partial x} + \rho v C_p \frac{\partial T}{\partial y} = y^{-j} \frac{\partial}{\partial y}\left(K y^j \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \rho D \sum C_{p_i} \frac{\partial Y_i}{\partial y}\frac{\partial T}{\partial y}. \quad (4)$$

Here,

$$j = \begin{cases} 0 \text{ if the flow is 2-}D\text{ symmetric} \\ 1 \text{ if the flow is axisymmetric.} \end{cases}$$

The turbulent transport coefficients to be determined are the turbulent diffusivity D, turbulent viscosity μ, and the turbulent thermal conductivity K. C_p is the specific heat at constant pressure.

Multiplying equation (1) by Y_i, equation (2) by y^j, adding the two, and then integrating the resulting equation with respect to y, one obtains after rearranging the terms

$$\rho D = \left[\int_0^y \frac{\partial}{\partial x}(\rho u Y_i) y^j \mathrm{d}y + \rho v Y_i y^j\right] \Big/ \left[y^j \frac{\partial Y_i}{\partial y}\right]. \quad (5)$$

Multiplying equation (1) by u, equation (3) by y^j, adding the two, and then integrating the resulting equation with respect to y, one obtains after rearranging the terms

$$\mu = \left[\int_0^y \frac{\partial}{\partial x}(\rho u^2) y^j \mathrm{d}y + \rho v u y^j\right] \Big/ \left[y^j \frac{\partial u}{\partial y}\right]. \quad (6)$$

Similarly, multiplying equation (1) by $C_p T$, equation (4) by y^j, adding the two, and then integrating the resulting equation with respect to y, one obtains after rearranging the terms

$$K = \left[\int_0^y C_p \frac{\partial}{\partial x}(\rho u T) y^j \mathrm{d}y + \int_0^y C_p \frac{\partial}{\partial y}(\rho v T y^j)\mathrm{d}y - \int_0^y \mu\left(\frac{\partial u}{\partial y}\right)^2 y^j \mathrm{d}y - \int_0^y \rho D\left(\sum C_{p_i}\frac{\partial Y_i}{\partial y}\frac{\partial T}{\partial y}\right) y^j \mathrm{d}y\right] \Big/ \left[y^j \frac{\partial T}{\partial y}\right]. \quad (7)$$

From equation (1),

$$v = -\frac{1}{\rho y^j}\int_0^y \frac{\partial}{\partial x}(\rho u) y^j \mathrm{d}y. \quad (8)$$

The values of the transport coefficients along the centerline or centerplane are obtained by substituting the above expression for v in equations (5)–(7) and then evaluating their limits as y approaches zero. There result

$$\lim_{y\to 0} \rho D = (\rho D)_{\mathbb{C}} = \frac{\rho_{\mathbb{C}} u_{\mathbb{C}} \dfrac{\partial Y_{2_{\mathbb{C}}}}{\partial x}}{(j+1)\left(\dfrac{\partial^2 Y_2}{\partial y^2}\right)_{\mathbb{C}}} \quad (9)$$

$$\lim_{y\to 0} \mu = \mu_{\mathbb{C}} = \frac{\rho_{\mathbb{C}} u_{\mathbb{C}} \dfrac{\partial u_{\mathbb{C}}}{\partial x}}{(j+1)\left(\dfrac{\partial^2 u}{\partial y^2}\right)_{\mathbb{C}}} \quad (10)$$

$$\lim_{y\to 0} K = K_{\mathbb{C}} = C_{p_{\mathbb{C}}} \frac{\rho_{\mathbb{C}} u_{\mathbb{C}} \dfrac{\partial T_{\mathbb{C}}}{\partial x}}{(j+1)\left(\dfrac{\partial^2 T}{\partial y^2}\right)_{\mathbb{C}}}. \quad (11)$$

Here, Y_2 is the mass concentration of the injected gas and was substituted for Y_i in equation (5) before arriving at equation (9). As $Y_2 = 1 - Y_1$, where Y_1 is the mass concentration of the primary gas, the diffusivity can also be expressed in terms of Y_1 simply by substituting Y_1 for Y_2 in equation (9). It is also pointed out here that due to symmetry about the centerline or centerplane

$$\left(\frac{\partial Y_i}{\partial y}\right)_{\mathbb{C}} = \left(\frac{\partial u}{\partial y}\right)_{\mathbb{C}} = \left(\frac{\partial T}{\partial y}\right)_{\mathbb{C}} = 0$$

which has been used in deriving equations (9)–(11).

Now, two variables P_l and Q_l, $l = 1, 2, 3$ are introduced such that

$$\begin{array}{ll} P_1 = Y_2 & Q_1 = \rho D \\ P_2 = u - u_e & Q_2 = \mu \\ P_3 = T - T_e & Q_3 = K/C_p \end{array}$$

where u_e and T_e are the values of velocity and temperature in the free stream. In the present case, both u_e and T_e are constant. Equations (9)–(11) can all be expressed now in the following form:

$$Q_{l_{\mathrm{C}}} = \frac{\rho_{\mathrm{C}} u_{\mathrm{C}} \dfrac{\partial P_{l_{\mathrm{C}}}}{\partial x}}{(j+1)\left(\dfrac{\partial^2 P_l}{\partial y^2}\right)_{\mathrm{C}}}, \qquad l = 1, 2, 3. \tag{12}$$

From experimental observations, it has been well established by now that in downstream regions of the wake

$$P_{l_{\mathrm{C}}} = \alpha_l x^{-m_l}, \qquad l = 1, 2, 3 \tag{13}$$

where α_l and m_l, $l = 1, 2, 3$ are constants, and

$$P_l/P_{l_{\mathrm{C}}} = f_l[y/\delta_l(x)], \qquad l = 1, 2, 3 \tag{14}$$

where δ_l is the proper wake half-radius defined as that value of y where $P_l = P_{l_{\mathrm{C}}}/2$. To be explicit, δ_1 is the concentration wake half-radius defined as that value of y where $Y_2 = Y_{2_{\mathrm{C}}}/2$; δ_2 is the velocity wake half-radius defined as that value of y where $u = (u_{\mathrm{C}} + u_e)/2$; and δ_3 is the temperature wake half-radius defined as that value of y where $T = (T_{\mathrm{C}} + T_e)/2$.

Now, from equation (13),

$$\frac{\partial P_{l_{\mathrm{C}}}}{\partial x} = -\alpha_l m_l x^{-m_l - 1} = -m_l \frac{P_{l_{\mathrm{C}}}}{x} \tag{15}$$

and from equation (14),

$$\left(\frac{\partial^2 P_l}{\partial y^2}\right)_{\mathrm{C}} = P_{l_{\mathrm{C}}} f_l''(0)/\delta_l^2 \tag{16}$$

where the prime indicates differentiation with respect to the variable $\eta = y/\delta_l(x)$. Substituting the above expressions for $\partial P_{l_{\mathrm{C}}}/\partial x$ and $(\partial^2 P_l/\partial y^2)_{\mathrm{C}}$ into equation (12), the following is obtained

$$Q_{l_{\mathrm{C}}} = -\left[\frac{m_l}{(j+1) f_l''(0)}\right] \rho_{\mathrm{C}} u_{\mathrm{C}} \delta_l^2/x. \tag{17}$$

Now, according to Forstall and Shapiro [6]

$$f_l = \left[1 + \cos\left(\frac{\pi}{2}\frac{y}{\delta_l}\right)\right]\Big/2 \tag{18}$$

from which

$$f_l''(0) = -\pi^2/8 = -1{\cdot}233$$

according to Townsend [7] and many others [3, 8, 9]

$$f_l = \mathrm{e}^{-(y/\delta_l)^2/2} \tag{19}$$

from which

$$f_l''(0) = -1$$

and according to Demetriades [10]

$$f_2 = \mathrm{e}^{-0{\cdot}43(y/\delta_2)^2} \text{ and } f_3 = \mathrm{e}^{-0{\cdot}34(y/\delta_3)^2} \tag{20}$$

from which

$$f_2''(0) = -0{\cdot}86 \text{ and } f_3''(0) = -0{\cdot}68.$$

Thus, it is seen that the absolute value of $f_l''(0)$ is always close to one. Hence, it can be safely assumed that for all practical purposes

$$f_l''(0) = -1, \qquad l = 1, 2, 3 \tag{21}$$

This reduces equation (17) to

$$Q_{l_{\mathrm{C}}} = \left(\frac{m_l}{j+1}\right) \rho_{\mathrm{C}} u_{\mathrm{C}} \delta_l^2/x, \qquad l = 1, 2, 3. \tag{22}$$

For analyzing the mixing process downstream in the wake, it may be assumed that at any axial station

$$Q_l = Q_{l_{\mathrm{C}}}. \tag{23}$$

Under the above assumption, the expressions for the turbulent transport coefficients are obtained from equation (22) by replacing Q_1, Q_2, and Q_3 therein with ρD, μ, and K/C_p respectively. The expressions are

$$\rho D = (\rho D)_{\mathrm{C}} = \left(\frac{m_Y}{j+1}\right) \rho_{\mathrm{C}} u_{\mathrm{C}} \delta_Y^2/x \tag{24}$$

$$\mu = \mu_{\mathrm{C}} = \left(\frac{m_u}{j+1}\right) \rho_{\mathrm{C}} u_{\mathrm{C}} \delta_u^2/x \tag{25}$$

$$K/C_p = (K/C_p)_{\mathrm{C}} = \left(\frac{m_T}{j+1}\right) \rho_{\mathrm{C}} u_{\mathrm{C}} \delta_T^2/x \tag{26}$$

where

$$\begin{aligned} m_Y &= m_1 & \delta_Y &= \delta_1 \\ m_u &= m_2 & \delta_u &= \delta_2 \\ m_T &= m_3 & \delta_T &= \delta_3 \end{aligned}$$

have been substituted in equations (24)–(26) respectively only for convenience so that the constant m_l and wake half-radius δ_l, $l = 1, 2, 3$ may easily be associated with mass concentration Y, velocity u, and temperature T respectively.

To compare the above expressions for D, μ and K with some other existing models for turbulent transport coefficients, the following models [1–4, 11] are mentioned:

1. Due to Zakkay *et al.* [1, 11]

$$\rho D = (\rho D)_{\mathrm{C}} = \frac{0{\cdot}6}{(1 + 0{\cdot}8\, W_j/W_e)(h/a)} \times \left(\frac{\rho_j u_j}{\rho_e u_e}\right)^{\frac{1}{2}} \rho_{\mathrm{C}} u_{\mathrm{C}} \delta_Y \tag{27}$$

where

$$h/a = 2 + 60(\rho_j/\rho_e)^{\frac{1}{2}} M_j^2 \left[1 + M_j^2\, \mathrm{e}^{-0{\cdot}5(M_e/M_j - 1)^2}\right]^{-2}.$$

Here, a = jet radius, h = concentration potential core length, M = Mach number, W = molecular weight, and the subscripts e and j denote conditions in the free stream and at the jet respectively.

$$\mu = \mu_{\mathbb{C}} = 0{\cdot}011\, \rho_{\mathbb{C}} u_{\mathbb{C}}\, \delta_u. \tag{28}$$

2. Due to Ferri *et al.* [2]

$$\mu = \mu_{\mathbb{C}} = 0{\cdot}025\, |\rho_e u_e - \rho_{\mathbb{C}} u_{\mathbb{C}}|\, \delta_{\rho u} \tag{29}$$

where $\delta_{\rho u}$ is the mass flow wake half-radius defined as that value of y where $\rho u = (\rho_e u_e + \rho_{\mathbb{C}} u_{\mathbb{C}})/2$, and $|F|$ denotes absolute value of F.

3. Due to Prandtl [1, 3, 4]

$$\mu = \mu_{\mathbb{C}} = k_1\, \rho_e\, |u_e - u_{\mathbb{C}}|\, \delta_u \tag{30}$$

where $k_1 = 0{\cdot}037$ for 2-*D* symmetric flow and $k_1 = 0{\cdot}025$ for axisymmetric flow.

4. Due to Schetz [4]

$$\mu = \mu_{\mathbb{C}} = k_2\, \rho_e u_e\, \delta^{*2}/a \tag{31}$$

where a is the initial radius of the jet, k_2 a constant to be determined experimentally, and δ^* a "displacement thickness" defined by the following relation:

$$\pi\, \rho_e u_e\, \delta^{*2} = \int_0^{\infty} |\rho_e u_e - \rho u|\, 2\pi\, y\, \mathrm{d}y. \tag{32}$$

It is observed that the above models [equations (27)–(31)] are all very different from the expressions for transport coefficients in equations (24)–(26), which have been derived here from the basic differential equations governing the turbulent mixing process and by taking into consideration the general behaviour of flow variables downstream in the wake. Moreover, none of these models show any direct dependence of the transport coefficients on the axial coordinate x as is seen in equations (24)–(26). This may explain the inadequacy of such simple models in predicting downstream flow fields [12, 13].

Values for the turbulent Schmidt number, Prandtl number, and Lewis number may now be easily obtained from the expressions for D, μ and K in equations (24)–(26) as the following:

$$S_c = \frac{\mu}{\rho D} = \left(\frac{m_u}{m_Y}\right)\left(\frac{\delta_u}{\delta_Y}\right)^2 \tag{33}$$

$$P_r = \frac{\mu C_p}{K} = \left(\frac{m_u}{m_T}\right)\left(\frac{\delta_u}{\delta_T}\right)^2 \tag{34}$$

$$L_e = \frac{\rho D C_p}{K} = \left(\frac{m_Y}{m_T}\right)\left(\frac{\delta_Y}{\delta_T}\right)^2. \tag{35}$$

It is pointed out here that the values of the constants m_Y, m_u and m_T are all known from experimental observations [1, 3, 8–12, 14–17]. For example, for axisymmetric turbulent wakes $m_Y = 2$ and $m_u = m_T = \frac{2}{3}$; and for 2-*D* symmetric turbulent wakes $m_Y = m_u = m_T = \frac{1}{2}$.

From the expressions for Schmidt number, Prandtl number, and Lewis number in equations (33)–(35), an interesting conclusion is drawn that the assumption of their values equal to unity may not be correct because in general $\delta_Y \neq \delta_u \neq \delta_T$. This has also been pointed out elsewhere on the basis of experimental observations [17] and numerical analysis [18].

III. CONCLUSIONS

Expressions for the turbulent diffusivity, viscosity, and thermal conductivity have been derived here from the basic differential equations governing the turbulent mixing of two dissimilar gases by taking into account the general behavior of flow variables downstream in the wake. 2-*D* symmetric and axisymmetric flows without chemical reactions have been considered. It is found that the transport coefficients are all proportional to $\rho_{\mathbb{C}} u_{\mathbb{C}} \delta^2/x$, where δ is the proper wake half-radius and the constant of proportionality is known. These values of the turbulent transport coefficients may be used to analyse the mixing processes and to predict the downstream flow fields. Moreover, it is further concluded that in general the assumption of Schmidt number, Prandtl number, and Lewis number equal to unity is not correct because the values of the wake half-radius for concentration, velocity and temperature are not necessarily the same.

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BOUNDARY EXTRAPOLATION IN HEAT TRANSFER CALCULATIONS

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NOMENCLATURE

B_i,	Biot number, hL/k or hr_0/k;
F_0,	Fourier number, $\alpha t/L^2$ or $\alpha t/r_0^2$;
h,	heat transfer coefficient;
J_0, J_1,	Bessel functions;
k,	thermal conductivity
L,	half-thickness of slab;
r_0,	radius of sphere or cylinder;
r, x,	coordinates;
R, X,	dimensionless coordinates r/r_0 or x/L;
t,	time;
T,	temperature.

Greek symbols

α,	thermal diffusivity;
β,	eigenvalues;
ε,	extrapolated distance;
θ,	dimensionless temperature, $(T - T_\infty)/(T_0 - T_\infty)$.

Subscripts

0,	initial condition;
∞,	ambient condition;
n,	order of eigenvalues;
ex,	exact;
ext,	extrapolated.

1. INTRODUCTION

THE TEMPERATURE distribution in heat transfer problems satisfies a differential equation. The differential equation